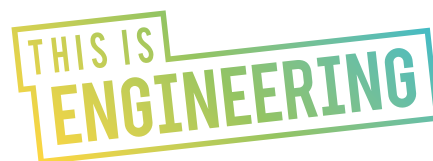


Royal Academy
of Engineering



A STEM learning and teaching resource that explores a variety of magical maths activities, from multiplication tips to card tricks.

Curriculum links
Maths: working mathematically, number, algebra.

Mind reading

Did you know that maths can help you to 'read minds'?

Amaze your friends by trying this trick. Using maths magic, you can guess their age and their shoe size, then impress them even more by telling them how the maths works!

Tell your friend to do the following things (but not to show you any calculations):

1. Write down your age.
2. Multiply it by $\frac{1}{5}$ of 100.
3. Add on today's date (e.g. 2 if it's the 2nd of the month).
4. Multiply by 20% of 25.
5. Now add on your shoe size (if it's a half size round down to a whole number).
6. Finally subtract 5 times today's date.
7. Show me your final answer!

Look at the answer. The hundreds are the age and the remaining digits are the shoe size. For instance, if somebody shows you 1105, there are 11 hundreds - the age, and the remaining digits 05 (or 5) show the shoe size.

Now, how on earth does that work? It isn't magic, it is algebra!

$x = \text{age}$ $y = \text{the date}$ $z = \text{shoe size}$

Explanation

Let's break it down into different parts:

Let's start with age (x):

Multiply your age (x) by $\frac{1}{5}$ of 100. $\frac{1}{5}$ of 100 is 20, so we have:

$$20x$$

Now we add today's date (y):

$$20x + y$$

Times it all by 20% of 25 (20% of 25 = 5):

$$5(20x + y) = 100x + 5y$$

Now we add the shoe size (z):

$$100x + 5y + z$$

Take away 5 times today's date ($5y$):

$$100x + 5y + z - 5y$$

We have $5y - 5y$ in the equation, so they disappear. Now we have:

$$100x + z$$

Example

So, let's do it with someone who is 11 years old and has shoe size 5, on 9 December:

1. The age is 11
2. $11 \times 20 = 220$
3. $220 + 9 = 229$
4. $229 \times 5 = 1145$
5. $1145 + 5 = 1150$
6. $1150 - (5 \times 9) = 1150 - 45 = 1105$
7. 11 is the age and 5 is the shoe size



Card magic

Step 1.

Deal out 27 cards face up into a grid of nine rows and three columns, like in the picture below.

Then put the rest of the cards to one side.



Step 2.

Ask a friend to pick a card, remember it and tell you only which of the three columns it is in.

Gather each of the columns back together so that you have three stacks face down, like this:



Step 3.

Pick up the stacks so that the one that contains your friend's card is on the bottom.

Step 4.

Now deal the cards again. Deal the same way as before, horizontally across the rows first. Again, ask them to tell you which column contains their card.

Step 5.

Pick up the columns vertically, this time making sure that the column containing the card is in the middle of the deck.

Step 6.

Finally, deal them out a third time in the same way, ask which column contains their card, and gather them up in the same way, with your friend's card in the middle deck.

Step 7.

Turn the cards over one by one but only count in your head. The 15th one will be your friend's card!

This seemed like pure magic but it's actually maths. This card trick is based on the ternary number system, sometimes called the base-3 system.

Due to the way you arranged and picked up the cards in stacks of three, your friend's card will always be in the 15th position, no matter what card they pick.

Explanation

To make the chosen card the 15th position in the deck, you need to have 14 cards above it.

To do this first need to express 14 in base-3:

$$14 = 112 \text{ in base } 3$$

For this trick, to work out how to arrange the cards we need to reverse the number, so this becomes 211.

With the understanding that:

2 = bottom

1 = middle

0 = top

The number 14 becomes "bottom-middle-middle."

This is why with the first deal, the card pile is placed on the bottom, then for the remaining two deals the pile is placed in the middle of the deck.

You can use this to extend the magic, and ask your friend to choose a card and their favourite number between 1 and 27, and use the number for the position of the card in the deck at the end.

This means you need between 0 and 26 cards on top of it, and in base 3 we have $0 = 000$ (top-top-top) and $26 = 222$ (bottom-bottom-bottom).

Every possible position that your friend can choose corresponds to a unique base-3 representation.

For a full explanation on this, watch the video: tinyurl.com/27cardtrick



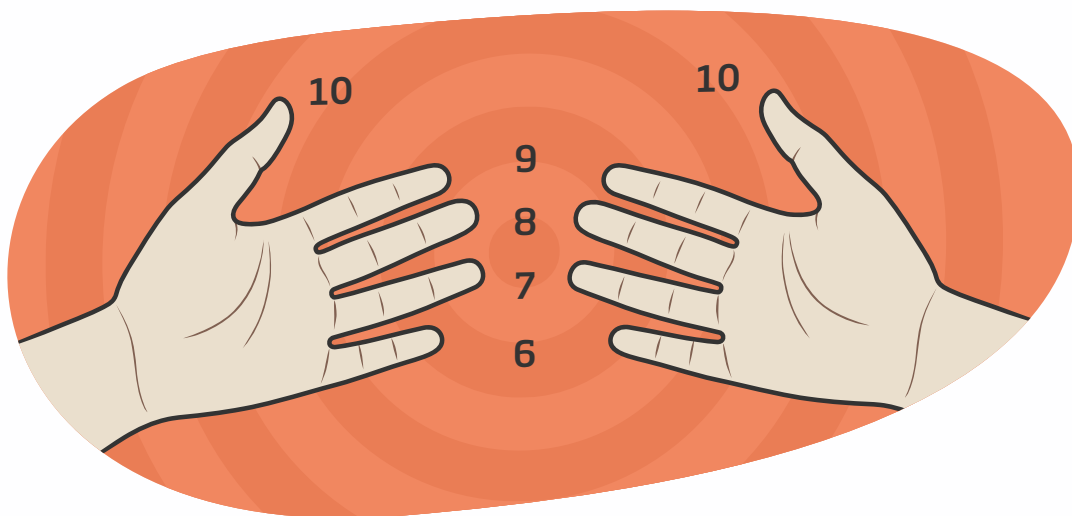
Magic hands

Did you know you can use your fingers as a magic 'short cut' to multiplication?

1. Hold your fingers out in front of you with your palms facing you.
2. In your head, label each finger with the following numbers:

your **thumb** is **10**,
index finger is **9**,
middle finger is **8**,
ring finger is **7** and
little finger is **6**.

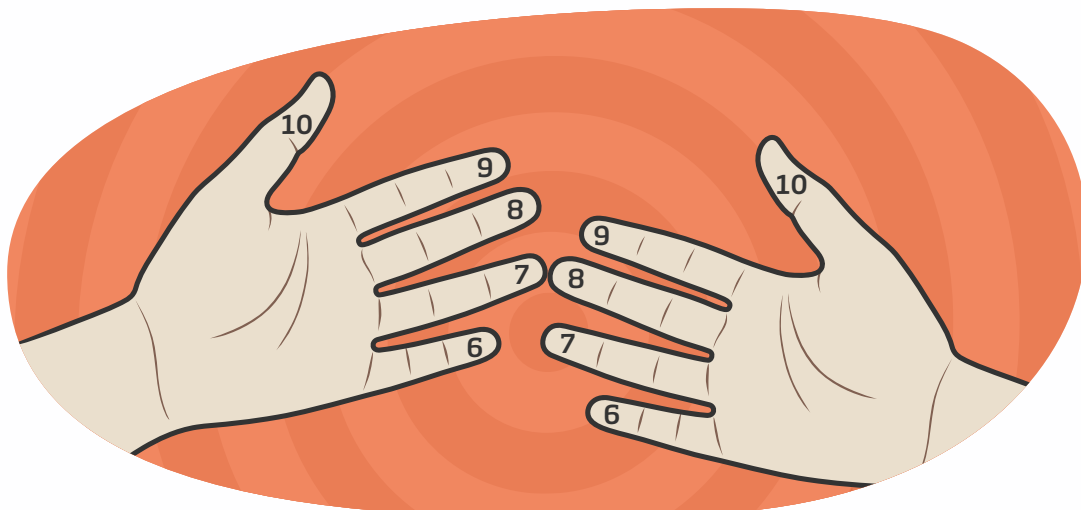
See below:



Now for the calculation! Let's work out 7×8 .

Step 1.

Find the finger that is **7** on your left hand, then the finger that says **8** on your right hand. Touch them together like this:



Step 2.

Look at the fingers above the ones that are currently touching.

Multiply the number of fingers above the ones that are currently touching on the left hand by the number above the ones that are currently touching the right hand.

On the left hand, there are 3 fingers (8, 9, 10)

On the right hand there are 2 fingers (8, 9)

Multiply 3 and 2 to make 6 ($3 \times 2 = 6$)

Step 3.

Look at the remaining fingers, the ones that are touching and the ones below.

Multiply the remaining number of fingers by 10

On the left hand, there are 2 fingers: the 7 and the 6

On the right hand, there are 3 fingers: the 8, the 7 and the 6

Now we add $2 + 3 = 5$

Multiply 5 by 10 to make 50.

Step 4.

We add the two numbers together to make 56.

Step 5.

We have the answer!

$$7 \times 8 = 56$$

Now try with these multiplications:

1. $6 \times 6?$
2. $7 \times 9?$
3. $8 \times 7?$
4. $9 \times 9?$

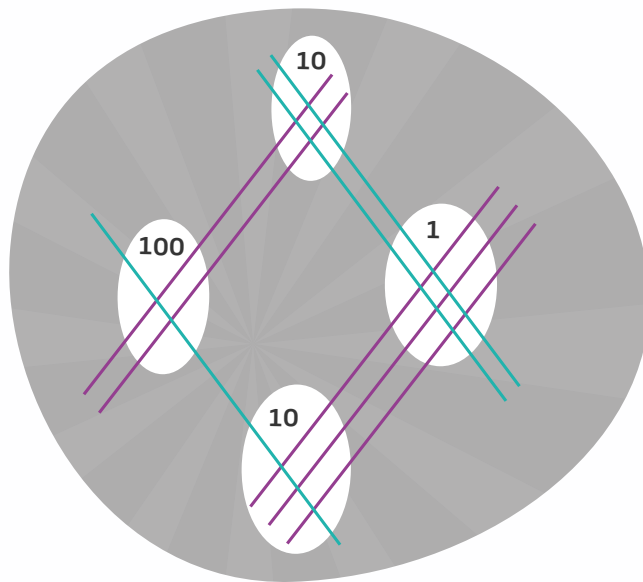


Magic multiplications

Now you can multiply up to the ten times table, but what about higher numbers, like 23×12 ?

For large multiplications, you can use the line method.

- Step 1.** Draw two lines slanted upward to the right.
- Step 2.** Move downward to the right a short distance draw another three lines upward to the right (the red lines in the image below). These lines represent 23.
- Step 3.** Then draw one line slanted downward to the right.
- Step 4.** Move upward to the right a short distance and draw another two lines slanted downward to the right (the green lines in the image below). These lines represent 12.
- Step 5.** Circle the corners of the square, where the lines meet. Label them with 100, 10 and 1 as shown below:



Now count the number of intersections in each corner of the figure.

The number of intersection points on the left (in the shaded area marked with 100) will be the number of hundreds in the answer. There are two intersections, so there are two hundreds in the answer.

The number of intersection points at the top and bottom of the square (in the shaded areas marked with 10) will be the number of tens in the answer. There are four intersections at the top, and three at the bottom, so there are seven tens in the answer.

The number of intersection points on the right (in the shaded area marked 1) will be the number of units in the answer. There are six intersections, so there are six units in the answer.

When you add these together the answer is 276.

Now try this method to calculate the following:

1. 12×24
2. 15×21
3. 25×25



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